LIST OF PROBLEMS

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We follow the notation in the page of open questions. For a survey of Galois points and related topics, see [18]. In what follows, comments and references are given in *italics* between *– and –*.

$\boxed{\text{I}}$ CHARACTERISTIC ZERO CASE

(A) Curve Case

(1) Galois points and Galois groups for plane curves C

(a) Find Galois points and the Galois groups for singular plane curves.
– for smooth curves, the number of Galois points is at most three (resp. four) if they are outer (resp. inner). The Galois groups are cyclic. [78, 95] –

(i) How is the structure of Galois group and how many Galois points do there exist? Is it true that the maximal number of outer (resp. inner) Galois points is three (resp. four)?
– *for rational curves* [73, 101, 102, 105], *for elliptic curves* [63], *for curves of prime degree* [12], *for non-immersed curves* [30] –

(ii) Study the property of singularity when C has a Galois point. In particular, if a singular point is also Galois, how is the property of the singularity? Find the characterization of the curve with the maximal number of Galois points.
– *for lower degree or rational curves* [69, 70, 71, 89, 101, 102, 105], *for two Galois points* [57] –

(iii) Does there exist a curve with three Galois points such that their groups are not isomorphic to one another? More generally, consider the set

$$\{ \ G_P \mid P \in \mathbb{P}^2 \text{ is a Galois point for } C\} \quad (\text{mod isomorphism}).$$

– [31, 64, 91] –

(iv) If C has a Galois point and its dual curve has one, what is the curve? Is it a self-dual curve?
– [44, 45] –

(b) Each element of G induces a birational transformation of C over $\mathbb{P}^1$. When is it extendable to a projective or birational transformation of $\mathbb{P}^2$?
– *for rational curves*, [73, 103] –
– *for relevant results*, [72, 74, 75, 77] –

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(c) Determine the group generated by $\{ \sigma \mid \sigma \in G_P,\ P \text{ is a Galois point}\}$ in the group of birational transformations of C.
– for a smooth curve and surface cases, [76] *and* [62]*, respectively –*
– the group generated by G_{P_1} and G_{P_2} is determined (in arbitrary characteristic), under the assumption that P_i is inner Galois and G_{P_i} fixes P_i for $i = 1, 2$ [68] *–*
– application to $\mathrm{Aut}(C)$ [55] *–*

(d) Classify plane curves in terms of the structure of the group generated by G_{P_1} and G_{P_2}, if P_1 and P_2 are Galois points.
– for the case where $G_{P_1}G_{P_2} = G_{P_1} \rtimes G_{P_2}$, [37, 48] *–*
– for the case where P_i is inner Galois and G_{P_i} fixes P_i for $i = 1, 2$, [68] *–*

(2) Non-Galois points for plane curves C

(a) Find the curve C with the constant Galois groups, i.e., for any $P \in \mathbb{P}^2$ the Galois group is the full symmetric group, in other words, if $P \notin C$ (resp. $P \in C$), then $G_P \cong S_d$ (resp. $G_P \cong S_{d-1}$).
– If P is a general point for C, then the Galois group at P is the full symmetric group of degree d (resp. $d-1$) if it is outer (resp. inner). [10, 54, 95] *–*

(b) Find a geometrical condition that G_P is primitive, i.e., the condition be given by the covering $\widetilde{\pi}_P : \widetilde{C} \longrightarrow \mathbb{P}^1$, where $\widetilde{C}$ is the normalization of C.
– for rational curves, [56] *–*

(c) Study these subjects in the remaining cases, i.e., the case where P is neither general nor Galois.

(i) Find the Galois group and the Galois closure curve for C at P, in particular the genus $g(P)$ of the Galois closure curve.
– for quartic curves, [78, 94] *–*

(ii) Determine the set
$\mathcal{G}(C) = \{\ g(P) \mid P \in C\ \}$ for a fixed smooth curve C.
In particular, determine $\mathcal{G}(F_d)$, where F_d is the Fermat curve of degree d.
Fixing d, determine the set
$\mathcal{G}(d) = \{\ g(P) \mid C \text{ is a smooth curve of degree } d \text{ and } P \in C\}$.
– for quartic curves, [94] *–*

(iii) Find the number of points at which the Galois groups are isomorphic to a fixed group. In particular, in the case where the group is an alternating group.
– for quintic curves [92]*, for dual curves of cubic curves* [45]*–*
– for relevant results [2, 8] *–*

(iv) Study the above in detail for special curves. For example, let F_d be the Fermat curve of degree $d \geq 5$. Suppose $d-1$ is not a prime number. Then, how is the Galois group at the flex?
– [79, 95] –

(3) Deformations of Galois closure curves

(a) For a smooth curve C consider the set $\{ \ C_P \mid P \in C \ \}$, where C_P is the Galois closure curve with respect to the projection $\pi_P : C \longrightarrow \mathbb{P}^1$.

(i) There exists a smooth projective surface S and a morphism $\varphi : S \longrightarrow C$ satisfying that $\varphi^{-1}(P) \cong C_P$, where P is a general point of C. Study the structure of S and the singular fiber of φ.
– [85, 86, 98] –

(ii) If a point P' is close to another one P, then are the Galois closure curves $C_{P'}$ and C_P not isomorphic to each other?
– [83, 85, 98] –

(b) Similarly, for a smooth curve C, consider the set $\{ \ C_P \mid P \in \mathbb{P}^2 \ \}$. There exists a smooth projective threefold M and a morphism $\psi : M \longrightarrow \mathbb{P}^2$ satisfying that $\psi^{-1}(Q) \cong C_Q$, where Q is a general point of $\mathbb{P}^2 \setminus C$. Study the structure of M and the singular fiber of ψ.
– [84] –

(4) Space curves

Study the same subjects for space curves. In particular, study the following:

(a) curves in $\mathbb{P}^3$

(i) Find Galois lines ℓ for C in two cases where $C \cap \ell = \emptyset$ and $C \cap \ell \neq \emptyset$. In particular, find the arrangement of Galois lines.
– *for quartic curves*, [13, 64, 106] –

(ii) Some space curve with a Galois line is obtained as a Galois closure curve of a plane curve. Characterize such a space curve.
– [99] –

(iii) Suppose C is a curve in $\mathbb{P}^3$ which is a complete intersection of two surfaces of degrees d_1 and d_2. Then, find the Galois lines and Galois groups. Does there exist any relation with the hypersurfaces?
– *for* $d_1 = d_2 = 2$, [106] –

(b) curves in $\mathbb{P}^n$

(i) Find the arrangement of Galois subspaces.
– *A general result is obtained in* [5] –

(ii) Study the Galois subspaces and Galois groups for a rational normal curve C. In particular, find the Grassmannian of the Galois subspaces.
– *This problem was solved in* [4] –

(iii) Study the Galois group when the subspace is not Galois.

(B) Surface and Hypersurface Cases

Study the same subjects for hypersurfaces. In particular, study the following:

(1) Galois points and Galois groups

   (a) Find the Galois points and Galois groups for hypersurfaces with singularities.
   – *for smooth hypersurfaces* [96, 97], *for normal quartic surfaces* [88], *for normal hypersurfaces* [49]. *It is proved that for any hypersurface (which is not a cone), the number of outer Galois points is finite* [9] –
   (b) Characterize the hypersurface with the maximal number of inner Galois points. Does it has a special property?
   – *quartic surfaces have some special property,* [62, 96] –

(2) Non-Galois points
   (a) When P is not a Galois point, consider the Galois closure surface.
   – *for the definition of L_W-normalization, see* [93] –
   – *In many cases the Galois closure surfaces are of general type. So, we have an interest in the case where they are not of general type.* [87] –

(C) Higher Dimensional Case and Galois Embedding

Study the same subjects for projective varieties V in $\mathbb{P}^N$. In order to treat the most general case, we should consider smooth varieties which are not necessarily in the projective space, so we consider the Galois embeddings.
– [100] –

(1) Study the Grassmannian

$$\{ \ W \in \mathbb{G}(N-n-1, N) \mid G_W \text{ is isomorphic to a full symmetric group } \}.$$

   In particular, is it true that the codimension of the complement of the set is at least two ?
   – [9, 11, 81] –
(2) Suppose $\dim \mathrm{Lin}(V) = 0$[‡], W is close to W' (in the Grassmannian) and $W \neq W'$. Let L_W be the Galois closure of the extension determined by the projection. Then, is it true that L_W is not isomorphic to $L_{W'}$?
(3) For an embedding (V, D) find the structure of Galois group G_W for each $W \in \mathbb{G}(N-n-1, N)$. In particular, let A be a principally polarized abelian variety with the polarization Θ. Then study the structure of A and the Galois group when $(A, 3\Theta)$ gives a Galois embedding.
   – *for an elliptic curve, the j-invariant is zero* –
(4) Find the structure of abelian variety if it possesses a Galois embedding.
   – *it is isogenous to the self product of an elliptic curve,* [3, 100] –
(5) Find the complex representation of the Galois group of abelian variety if it possesses a Galois embedding.
   – *for abelian surfaces,* [100] –
(6) For each nonsingular projective algebraic variety V, consider whether it possesses a Galois embedding.

[‡]Denote by $\mathrm{Lin}(V)$ the subgroup of $\mathrm{Aut}(V)$ consisting of elements induced by the projective transformations of the ambient space which leave V invariant.

– *for self products of smooth curves,* [3] –

(7) Find all the Galois subspaces for one Galois embedding, or find the arrangements of Galois subspaces. Suppose the Kodaira dimension of V is non-negative. Then, is it true that the number of Galois subspaces is finitely many?
– [99] –

(8) For the surface with Kodaira dimension ≤ 0, consider the Galois embeddings, the Galois subspaces and the Galois groups.
– [100, 104, 108, 111] –
– *bielliptic surfaces have no Galois embeddings* [109] –

(9) Some projective variety with a Galois subspace is obtained as a Galois closure variety of another projective variety. Characterize such a variety.
– [110] –

(10) For each finite subgroup G of $GL(2, k)$, does there exist a pair (V, D) which defines the Galois embedding with the Galois group G such that $D^n = |G|$, $\dim V = n$ and $\dim \mathrm{H}^0(V,\ \mathcal{O}(D)) = n + 3$?
– [110] –

(11) Consider the similar subjects in the case where $f(V) \cap W \neq \emptyset$.

(D) Related Topics

(1) Let $k(x, y)$ be an algebraic function field of one variable over k. Suppose $k(x, y)/k(x)$ is a Galois extension and σ a Galois automorphism. Then, how can we express $\sigma(y)$ as an element of $k(x, y)$?
– *this may have some relation with the singularity of the curve defining* $k(x, y)$, [107] –

(2) In case the curve C is defined over $\mathbb{Q}$, can we develop the similar research? How is the Galois group at rational points? If the "degree of a point" becomes large, then how does the Galois group at the point become? Suppose C has good reduction C_p modulo p. Then, compare the Galois groups at the points in C and C_p.
– *Theorem 3 in* [95] –

(3) When an irreducible curve C exists in a ruled surface S, consider the projection $\pi : S \longrightarrow \Delta$, where Δ is a base curve. Suppose C is not a fiber of π. Then, restricting π to C, we get an extension of function fields $k(C)/\pi^* k(\Delta)$. Do the similar research for this case as in the plane curve case. If C is smooth and $\pi|_C$ a Galois cover, then is the Galois group cyclic?
– *Yes, for the Hirzebruch surface* $S = \Sigma_n$ $(n \geq 1)$, *see for the details and other results,* [90] –

(4) Study the same subject as above in a weighted projective space, i.e., consider a weighted projective variety, projection and function field ...

(5) Extend the study of Galois points to that of "quasi-Galois points".
– [46, 47] –

(6) Study the relations between Galois points and weak Galois Weierstrass points.
– [66, 67] –

$\boxed{\text{II}}$ POSITIVE CHARACTERISTIC CASE

(A) Throughout the case where $p > 0$: Generalize results obtained in the case where $p = 0$ to arbitrary characteristic $p \geq 0$.

– *We have not checked yet whether a lot of results on Galois points obtained in the case where $p = 0$ hold also in $p > 0$. Therefore, almost all problems in* $\boxed{\text{I}}$ *are open also in this case.* –

(B) Curve Case

(1) Galois points and Galois groups for plane curves C

(a) Find Galois points and the Galois groups for singular plane curves.
– *for smooth curves, the number of Galois points and the Galois groups were determined.* [14, 15, 16, 19, 21, 22, 59] –

(i) Find new examples of plane curves having many Galois points. Determine the Galois groups for such curves.
– *see tables in* $\boxed{\text{III}}$ –

(ii) Give an upper bound for the number of Galois points if the number is finite, and characterize curves attaining the bound. If the number of outer Galois points is finite and at least $(d-1)^4 - (d-1)^3 + (d-1)^2$, then is such a curve Hermitian? What is the next upper bound?
– *for inner Galois points, the upper bound is* $(d-1)^3 + 1$ [23, 27] –

(iii) Find new examples of curves admitting non-collinear Galois points. Is it true that rational, elliptic, Fermat, Hermitian and Dickson–Guralnick–Zieve curves are all (smooth models of) curves admitting non-collinear Galois points?

(iv) Classify curves admitting non-collinear Galois points. Is it true that if there exist non-collinear outer Galois points P_1, P_2 and P_3 and points Q_1, Q_2 and $Q_3 \in C$ such that $\overline{P_iP_j} \ni Q_k$ and $\langle G_{P_i}, G_{P_j} \rangle Q_k = Q_k$ for any i, j, k with $\{i, j, k\} = \{1, 2, 3\}$, then (the smooth model of) C is rational or the Hermitian curve?
– *cf.* [33] –

(v) Find Galois points and Galois groups for special curves in positive characteristic. For example, when C is a non-reflexive or strange curve.
– *see* [61, 65] *for definition. for a non-reflexive curve of low degree* [20] –

(vi) Find Galois points and the Galois groups for singular curves of lower genera. For rational curves not defined by $x - y^q = 0$, is it true that the number of inner (smooth) Galois points is at most d?
– *Even if the genus* (*of the smooth model*) *is* 0 *or* 1*, this problem is still open in the case where* $p > 0$. –

(vii) Classify plane curves with many singular points which are Galois. Is there a plane curve of genus g such that the number of such Galois points is $(d-1)(d-2)/2 - g$?

– *Such a rational curve exists* [20]. *The* (q^3, q^2)*-Frobenius nonclassical curve has many singular points which are Galois* [6]. –

(b) A birational embedding of an algebraic curve X into $\mathbb{P}^2$ with two or more Galois points (C is the image of the embedding of X)

(i) Find a criterion for the existence of a birational embedding with three collinear Galois points.
– *for a birational embedding with two Galois points* [31], *for a birational embedding with non-collinear Galois points* [33]. *One version for three collinear Galois points is proposed in* [34]. –

(ii) For the genus $g \geq 2$, determine a function $f(g)$ such that if a curve X has an automorphism group G with $|G| \geq f(g)$ then X has a birational embedding with (two) Galois points.
– *This is similar to a problem posed in* [52]. *Compared to* [32, 39, 59], *it follows from Stichtenoth–Henn's classification* [58, Theorem 11.127] *that* $f(g) \leq 8g^3$. –

(iii) Describe the full automorphism group $\mathrm{Aut}(X)$ of an algebraic curve X possessing a birational embedding into $\mathbb{P}^2$ with (two or more) Galois points.
– *cf.* [1, 7, 36, 68, 80] –

(iv) Determine all birational embeddings with two (or more) Galois points when X is fixed. How about the Hermitian curve?
– *The Hermitian curve has three kinds of such embeddings* [32, 33, 59]. –

(2) Non-Galois points for plane curves C
– *In positive characteristic, the Galois group* G_P *at a general point* $P \in \mathbb{P}^2 \setminus C$ *(resp.* $P \in C$*) is not always the full symmetric group, even if* C *is smooth.* [60] –

(a) If C is reflexive, is the Galois group G_P at a general point $P \in \mathbb{P}^2 \setminus C$ (resp. $P \in C$) the full symmetric group?
– *cf.* [81, 82] –

(b) If C is non-reflexive, what kind of group appears as the Galois group G_P at a general point $P \in \mathbb{P}^2 \setminus C$ (resp. $P \in C$)? What is the genus $g(P)$ of C_P?

(c) Classify curves C with two points P_1 and $P_2 \in \mathbb{P}^2$ such that $L_{P_1} = L_{P_2}$, or the Galois closure curves.
– *Such curves are studied in* [42]. –

(3) Space curves
Describe the arrangement of Galois lines for curves in $\mathbb{P}^3$. How about special curves in positive characteristic?
– *for the Giulietti–Korchmáros curve* [40], *for the Artin–Schreier–Mumford curve and generalized curves* [35, 36] –

(4) Study the relations between Galois points and other subjects of research.

(a) Study birational embeddings of maximal curves, which attain the Hasse–Weil bound, into $\mathbb{P}^2$ with Galois points.
– [32, 39, 40, 41] –

(b) Do there exist any relations between Galois points and the p-rank of C? Find Galois points for curves with p-rank zero.
– [7, 26] –

(c) Study the relations between Galois points and rational points when C is defined over a finite field. If the set of Galois points coincides with that of rational points of $\mathbb{P}^2$ (over a finite field), is it true that the curve C is the Hermitian, Ballico–Hefez or the (q^n, q^m)-Frobenius nonclassical curve of type $(n, m) = (3, 1)$ or $(3, 2)$? Or, more basically, is C Frobenius nonclassical?
– [6, 19, 20, 28, 59] –

(d) Is there an application to Coding theory? Find a curve C defined over a finite field whose Galois points are rational, and study algebraic-geometric codes from C with such points.
– *cf.* [43] –

(e) Study the relations between Galois points and complete arcs arising from (Frobenius nonclassical) plane curves.
– *see* [53] *for the definition of complete arcs arising from plane curves. for a certain abelian p-cover of the Hermitian curve,* [7] –

(f) Is there an application to Cryptography?

(g) Is there an application to Group theory?
– [50] –

(C) Hypersurface Case

(1) Find Galois points for smooth or normal hypersurfaces.
– *The Galois groups at Galois points have been determined* [49]. *For the Fermat hypersurface of degree* $p^e + 1$*, the distribution of Galois points is determined.* –

(2) Classify hypersurfaces with infinitely many Galois points.
– *for the case where the dimension of the set of Galois points is equal to that of the hypersurface* [24] –

(3) Find new examples of hypersurfaces having many Galois points. Determine the Galois groups for such hypersurfaces.

(4) Find Galois points and the Galois groups for special hypersurfaces in positive characteristic. For example, non-reflexive or strange hypersurfaces.
– *The Fermat hypersurface of degree* $p^e + 1$ *has many Galois points and is non-reflexive.* –

(5) Study the relations between Galois points and rational points for a hypersurface.

(D) Higher Dimensional Case

(1) Classify projective varieties with infinitely many Galois subspaces.
(2) Find Galois subspaces and the Galois groups for special varieties in positive characteristic. For example, non-reflexive or strange varieties.
– *The studies of non-reflexivity from a certain viewpoint are discussed in* [61]. –
(3) Study the relations between Galois subspaces and rational points for a projective variety.

$\boxed{\text{III}}$ APPENDIX: Tables of plane curves with two or more Galois points

(Update 19 April, 2021 for $\delta'(C)$; 26 February, 2019 for $\delta(C)$)

We denote by $\delta'(C)$ (resp. $\delta(C)$) the number of Galois points which is contained in $\mathbb{P}^2 \setminus C$ (resp. $C \setminus \mathrm{Sing}(C)$). If the characteristic p is positive, then we assume that q is a power of p.

Summarizing results obtained by several authors by now, we make tables of plane curves with $\delta'(C) \geq 2$ and with $\delta(C) \geq 2$. In the tables, "groups" mean groups appearing as Galois groups at Galois points and "elem. p" means an "elementary abelian p-group $(\mathbb{Z}/p\mathbb{Z})^{\oplus e}$". (Remark, (*1)(*2): There exist families of curves with $\delta'(C) \geq 2$ to which these curves belong ([21, 25]).)

	$\delta'(C)$	p	d	curve	groups	ref.
(1)	∞	>0	p^e	$\sum_{i=0}^{e}(\alpha_i x^{p^i}+\beta_i y^{p^i})=0$	elem. p	[38, 17]
(2)	$q^4-q^3+q^2$	>0	$q+1$	Fermat curve	cyclic	[59]
(3)	$q(q+1)/2$	>0	$q+1$	$(1:(1+t)^{q+1}:t^{q+1})$	cyclic	[20]
(4)	$q+1$ or $q-1$	>0	$2q$	$(x^q-x)^2+(x^q-x)(y^q-y)$ $+\lambda(y^q-y)^2+\mu=0$ (*1) $(\lambda\in\mathbb{F}_q,\ (q,\lambda,\mu)\neq(2,1,1))$	$(\mathbb{Z}/p\mathbb{Z})^{\oplus e}$ $\rtimes$ $\mathbb{Z}/2\mathbb{Z}$	[21, 25]
(5)	q^2+q+1	>0	q^3-q^2	Dickson–Guralnick–Zieve	$\mathbb{F}_q^{\oplus 2}\rtimes\mathbb{F}_q^{\times}$	[19, 6]
(6)	q^2-q	>0	$q^{2n}(q+1)$	$(\sum_{i=0}^{n}\alpha_i x^{q^{2i}})^{q+1}+(\sum_{i=0}^{n}\alpha_i y^{q^{2i}})^{q+1}$ $+c=0$	$\mathbb{F}_{q^{2n}}\rtimes$ $\mathbb{Z}/(q+1)\mathbb{Z}$	[7]
(7)	3	≥ 0	$\not\equiv 0 \bmod p$ $\neq q+1$	Fermat curve	cyclic	[78, 95]
(8)	3	0		$(1:(1+t)^d:t^d)$	cyclic	[101]
(9)	≥ 3	>0	$s(q+1)$ $s\mid q-1$	birational embedding of Hermitian	cyclic	[33]
(10)	2	≥ 0	$2m$ $\not\equiv 0 \bmod p$	$x^{2m}+x^m+y^{2m}=0$	cyclic dihedral	[91, 48]
(11)	≥ 2	$\neq 2$	8	$x^2(x^2+1)(x^2+\frac{1}{2})^2+y^8=0$	cyclic dihedral	[48]
(12)	≥ 2	$\neq 2,3$	6	$(x^2+1)(x^2+\frac{1}{4})^2+y^6=0$	cyclic dihedral	[48]
(13)	≥ 2 $=2$ (many cases)	>0	$q\ell$ $p\nmid\ell,\ \ell\geq 3$ $\ell\mid q-1$	$(x^q-x)^\ell+\lambda(y^q-y)^\ell$ $+\mu=0$ (*2)	$(\mathbb{Z}/p\mathbb{Z})^{\oplus e}$ $\rtimes$ $\mathbb{Z}/\ell\mathbb{Z}$	[25]
(14)	≥ 2	>0	$q+1$	birational embedding of $y^m=x^q+x$ $(m\mid q+1)$	cyclic	[39]
(15)	≥ 2	>0	q^r+1	birational embedding of $y^{q^r+1}=x^q+x$	cyclic	[39]
(16)	≥ 2	≥ 3	$q+1$ >6	$(t^{\frac{q+1}{2}}:(t+1)^{\frac{q+1}{2}}(t+\gamma)^{\frac{q+1}{2}}:t^{q+1}-\gamma)$ $(\gamma\in\mathbb{F}_q\setminus\{\pm1\},\ \gamma^{\frac{q-1}{2}}=1)$	dihedral	[50]
(17)	2	≥ 3	$q+1$ >4	$(t^{\frac{q+1}{2}}:(t+1)^{q+1}:t^{q+1}+\gamma)$ $(\gamma\in\mathbb{F}_q\setminus\{0,\pm1\})$	cyclic dihedral	[50]
(18)	≥ 2	11	12	birational embedding of $\mathbb{P}^1$	cyclic, dihedral, or A_4	[51]
(19)	≥ 2	23	24	birational embedding of $\mathbb{P}^1$	cyclic, dihedral, or S_4	[51]
(20)	≥ 2	59	60	birational embedding of $\mathbb{P}^1$	cyclic, dihedral, or A_5	[51]

	$\delta(C)$	p	d	curve	groups	ref.
(1)	∞	>0	q	$x - y^q = 0$	cyclic	[38]
(2)	q^3+1	>0	$q+1$	Fermat curve	elem. p	[59]
(3)	$q+1$	>0	$q+1$	$(1 : (1+t)^{q+1} : t^{q+1})$	elem. p	[20]
(4)	$q+1$	2	$q+1$	$\prod_{\alpha\in\mathbb{F}_q}(x+\alpha y+\alpha^2)+cy^{q+1}=0$ $(c \neq 0,1)$	elem. p	[19, 22]
(5)	$q+1$	>0	q^3+1	(projected) Giulietti–Korchmáros	p-group	[40]
(6)	4	$\neq 2,3$	4	$x^3+y^4+1=0$	cyclic	[78]
(7)	3 or 2	$\neq 3$	4	$((t+\alpha)^3 : t(t+\beta)^3 : t(t+1)^3)$ $(\beta^4 \neq \beta,\ \alpha = (\beta^2+\beta+1)/3)$	cyclic	[71, 29]
(8)	≥ 2	$\neq 3$	4	birational embedding of Fermat cubic	cyclic	[31]
(9)	≥ 2	$\neq 2,3$	5	birational embedding of $\mathbb{P}^1$	cyclic $(\mathbb{Z}/2\mathbb{Z})^{\oplus 2}$	[31]
(10)	2	$\neq 2,5$	6	birational embedding of $\mathbb{P}^1$	cyclic	[31]
(11)	2	$\neq 2,3$	4	$(1 : (1+t)^3 : t^4)$	cyclic	[29]
(12)	2	>0	q^3+1	birational embedding of Hermitian	p-group	[32]
(13)	2	>0	$q+1$	birational embedding of $y^m = x^q + x$ $(m \mid q+1)$	elem. p	[39]
(14)	2	≥ 3	q	$(t^{\frac{q+1}{2}} : (t-1)^{\frac{q+1}{2}} : t^q - t)$	dihedral	[50]
(15)	2	≥ 3	q	$(t^{\frac{q+1}{2}} : t-1 : t^q - t)$	cyclic dihedral	[50]
(16)	2	3	q^3+1	birational embedding of Ree	p-group	[32]
(17)	2	2	q^2+1	birational embedding of Suzuki	p-group	[32]
(18)	≥ 2	>0	q^3+1	quotient curves of GK curve	p-group	[41]
(19)	≥ 2	3	q^3+1	(quotients of) Skabelund curve of Ree type	p-group	[41]
(20)	≥ 2	2	q^2+1	(quotients of) Skabelund curve of Suzuki type	p-group	[41]
(21)	≥ 2	≥ 0	5	birational embedding of an elliptic curve		[68]
(22)	≥ 2	≥ 0	7	birational embedding of an elliptic curve		[68]
(23)	≥ 2	2	25	birational embedding of an elliptic curve		[68]

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