Introduction to Galois Point

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1. transcendental extension of field
2. introduction to Galois Point
3. examples and theorems.

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Preliminary Remark 1

First, recall the following:
it is well-known that

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

can be expressed by elementary function $(= sin^{-1}x)$, but

$$\int \frac{1}{\sqrt{1-x^3}}\,dx$$

can not.
Why ?
What is the essential difference between them?

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Preliminary Remark 2

The difference is
the former is given by $y = \sqrt{1 - x^2}$, i.e., $x^2 + y^2 = 1$ and
the latter is given by $y = \sqrt{1 - x^3}$, i.e., $x^3 + y^2 = 1$.
The former curve can be parametrized by rational functions

$$x = \frac{1 - t^2}{1 + t^2}, \quad y = \frac{2t}{1 + t^2}.$$

Whereas, the latter cannot.
In fact, the inverse of the integral is an elliptic function.
Let us examine this in more detail.

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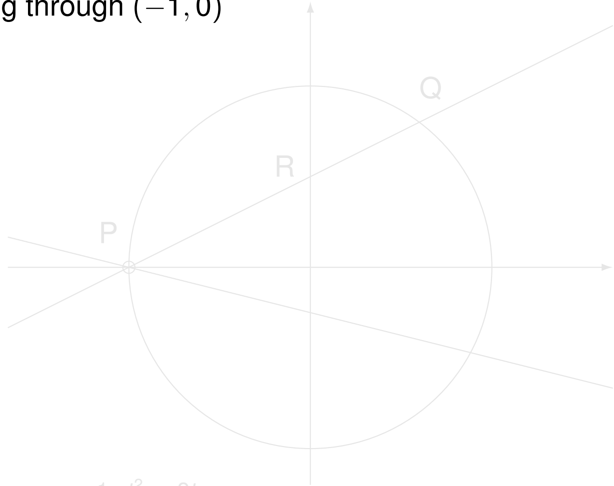
Preliminary Remark 3

Consider the circle $x^2 + y^2 = 1$ and the lines $y = t(x + 1)$ passing through $(-1, 0)$



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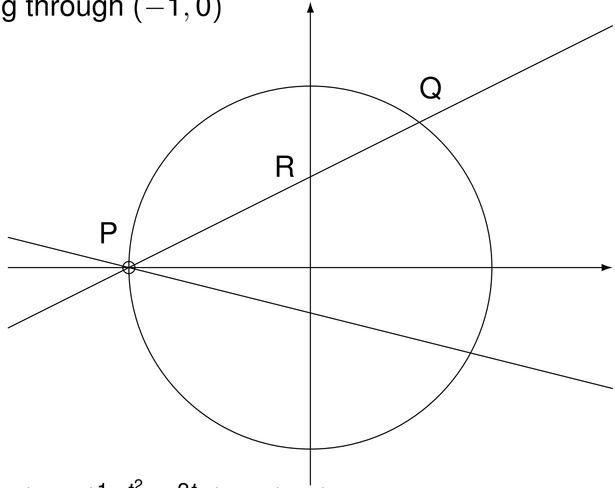

$P(-1, 0),\ Q(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}),\ R(0, t)$

Preliminary Remark 4

There is a mapping $\pi_P : C \setminus \{P\} \longrightarrow L$, where L is the y-axis and $\pi_P(Q) = R$. This mapping is one of a projection.
This is a 1:1 correspondence and induces an isomorphism between C and projective line.
So the circle can be parametrized by a line.
The function field of circle is the quotient field of the domain $k[x, y]/(x^2 + y^2 - 1)$.
It is rational, i.e., it is isomorphic to $k(t)$, i.e., purely transcendental extension over a ground field.
Indeed, $k(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}) = k(t)$, so the circle is a rational curve.
By the similar way we can parametrize quadratic curves by rational functions.
So the quadratic curve is a rational curve.,

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Preliminary Remark 5

The curve $x^3 + y^2 = 1$ can not be parametrized by any rational function.

(Prove this as an exercise!)

Note that rational or not is not determined by the degree.

It's determined by genus of the curve.

For example, the curve $y^2 = x^2(x+1)$ is a rational curve,

Indeed, take the projection center at $(0,0)$.

Namely consider the line $y = tx$ passing through $(0,0)$.

Then we have $x = t^2 - 1, y = t(t^2 - 1)$.

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Transcendental Extension 1

Please allow me to state my personal interest.
I had an interest in field theory.
Suppose K is a field finitely generated over a field k.
If the extension K/k is algebraic, then there are several methods to study the property of the extension,
for example, degree, Galois theory etc.
However, if is is transcendental, then there are few suitable ones.
How to study the extension K/k?
I think purely transcendental extension is the most simple one.
So, I take the purely transcendental extension as the starting point.

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Transcendental Extension 2

Let n be the transcendental degree.

In this case, we pay attention to a maximal rational subfield K_m, which has the following properties:

1. K_m is an intermediate field between K and k,
2. and purely trans. ext. of k with the trans. degree n.
3. there is no field between K and K_m.

   Then, we consider the algebraic extension K/K_m

   However, there is an inconvenient point.

   In fact, even if $n = 1$ and $K = k(x)$, there are many maximal rational subfields:

   $k(x^2),\ k(x^3), \ldots,\ k(x^p), \ldots$ (p is a prime number)

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2. and purely trans. ext. of k with the trans. degree n,
3. there is no field between K and K_m.
   Then, we consider the algebraic extension K/K_m
   However, there is an inconvenient point.
   In fact, even if $n = 1$ and $K = k(x)$, there are many maximal rational subfields:
   $k(x^2),\ k(x^3), \ldots,\ k(x^p), \ldots$ (p is a prime number)

Transcendental Extension 2

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Irrationality

So, we use the notion: the degree of irrationality, which is defined as follows:

$\min\{\ [K:K_m]\ :\ K_m$ is a maximal rational subfield. $\}$

We denote this number by $\mathrm{irr}(K/k)$ or $\mathrm{irr}(K)$.

The field K is rational if and only if $\mathrm{irr}(K) = 1$.

Maximal rational subfield F with $[K:F] = \mathrm{irr}(K)$ is called g-maximal rational subfield.

For example, for the elliptic function field

$k(x,\ y),\ \ y^2 = x^3 + ax + b,\ \ 4a^3 + 27b^2 \neq 0$

$\mathrm{irr}(k(x,\ y)) = 2$ and $k(x)$ is a g-maximal rational subfield, $k(y)$ is a maximal rational field but not a g-maximal one.

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Algebraic Function Field 1

I mention one more fact

On an algebraic variety there exist natural functions.

If R is a coordinates ring of an affine part, then an element of the quotient field of R is the natural function.

The field is an algebraic function field.

The transcendence degree coincides with the dimension of the variety.

A field extension L/K corresponds to a dominant rational map $f : V \dashrightarrow W$,

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Algebraic Function Field 2

If the extension is algebraic, then the morphism is a covering.
Moreover, if the extension is Galois, then the covering is Galois.
So, we can "see" the field extension by the mapping between varieties.
For example, for the elliptic function field $k(x, y)$, the extension $k(x, y)/k(x)$ corresponds to
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Motive

I explain the motive of this research.
Roughly speaking, algebraic variety is a realization of algebra.
Commutative ring R corresponds the scheme Spec(R).
So, we can study variety by algebra, and vice versa.
Let's take up again the elliptic curve $y^2 = x^3 + 1$.
It is not simple to prove that the curve is parametrized by rational function.
However it is simple by using geometry.
We have only to check the existence of regular form.
In fact, the form $\frac{dx}{2y} = \frac{dy}{3x^2}$ is a regular 1-form.
The genus is not zero, so that the curve is not rational.

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Before proceeding to the definition of Galois point, I use a simile here.

Very rough consideration before the definition:

Suppose there is a cube in a space. Look at it from several positions by one eye.

In general we cannot find any symmetry in the figure.

However, at some special points it looks like a regular hexagon or a square.

We seek for such the special points for algebraic curves in the projective plane.

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Galois Point 1

Hereafter we discuss on the ground field k, which is assumed to be an algebraically closed field of characteristic zero.

Let C be an irreducible projective plane curve of degree d and $k(C)$ the function field.

Let P be a point in the plane $\mathbb{P}^2$
and consider the projection $\pi_P : \mathbb{P}^2 \dashrightarrow \mathbb{P}^1$ with the center P.

Restricting π_P to C, we get a dominant rational map
$\bar{\pi}_P : C \dashrightarrow \mathbb{P}^1$,
which induces a finite extension of fields $\bar{\pi}_P^* : k(\mathbb{P}^1) \hookrightarrow k(C)$ of degree $d - m$,
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Let P be a point in the plane $\mathbb{P}^2$
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Galois Point 2

We assume $d - m \geq 3$.
If the extension is Galois, we call P a Galois point for C.
If, moreover, $P \in C$ [resp. $P \notin C$], then we call P an inner [resp. outer] Galois point.
We denote by $\delta(C)$ and $\delta'(C)$ the number of inner and outer Galois point, respectively.
Let $G = G_P$ be the Galois group.
By definition an element of G induces a birational transformation of C over the projective line $\mathbb{P}^1$.
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Moreover, if $d \geq 4$, then the automorphism can be extended to a projective transformation of $\mathbb{P}^2$

and hence G turns out a cyclic group .

The $\bar{\pi}_P$ induces an extension of fields $k(C)/k(\bar{\pi}^*(\mathbb{P}^1))$ of degree $d-1$ or d,

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In case the extension $k(C)/k(\mathbb{P}^1)$ is not Galois,
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Way to find

Generally, it is difficult to find Galois point for given curve. The following is a way to find all the Galois points.
First, find the flexes by using Hessian of the curve C.
By this we have the candidates of Galois point.
From them determine Galois point, by considering if the extension is Galois. .
If Q is a flex, we have

$$W(C) = \sum_{Q \in C} \{i(C, T_Q; Q) - 2\} = 3d(d-2)$$

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where T_Q is the tangent line to C at Q,
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Example 1

We present a special example.
For the curve $YZ^3 + X^4 + Y^4 = 0$, find all Galois points.
The Hessian is $2^2 3^3 X^2 Z(8Y^3 - Z^3)$.
We infer from this there are 20 flexes.
Since $W(C) = 24$, there exist at most four Galois points on $X = 0$.
For example, at $P = (0:0:1)$, putting $x = X/Z, y = Y/Z$, each line passing P is $y = tx$.
The defining equation is $y + x^4 + y^4 = 0$, so that $(1 + t^4)x^3 + t \in k(t)[x]$.
This gives a Galois extension over $k(t)$, and the Galois group is the cyclic group of order three.
Galois automorphism is extended to the action
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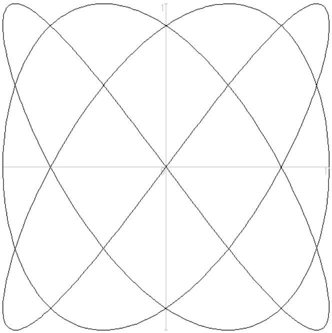
The defining equation is $y + x^4 + y^4 = 0$, so that $(1 + t^4)x^3 + t \in k(t)[x]$.
This gives a Galois extension over $k(t)$, and the Galois group is the cyclic group of order three.
Galois automorphism is extended to the action $(X : Y : Z) \to (\omega X : \omega Y : Z)$
where ω is a primitive cubic root of 1.
All the inner Galois points are $(0, 0)$, $(0, \alpha)$, where $\alpha^3 = -1$

Lissajous's Curve ($x = \cos 3\theta$, $y = \sin 4\theta$)



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Let us examine classically familiar curves.

Let $m, \; n$ be coprime positive integers. Put $d = 2\max\{m, n\}$

Let $S(m, n)$ and $CS(m, n)$ be the complexified curves defined by

$x = \sin m\theta, \; y = \sin n\theta$ and $x = \cos m\theta, y = \sin n\theta$,

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They are called Lissajous's curves.

Miura found the following:

(1) $S(m, n)$ has a D_d-point if $m + n$ is odd,

A point P is called a D_d-point if it is a Galois point with Galois group isomorphic to the dihedral group of order d.

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Putting $\xi = Z/X$, $\eta = Y/X$ and $t = \exp\sqrt{-1}\theta$, we get

$-16X^6Z^2 + 80X^4Z^4 - 128X^2Z^6 + 64Z^8 + 9X^6Y^2 - 24X^4Y^4 + 16X^2Y^6 = 0.$

This curve is a rational curve with only double points as the singularities
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Moreover, let $\varphi : \mathbb{P}^1 \hookrightarrow \mathbb{P}^8$ be
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Let $(T_0, T_1, \cdots, T_8)$ be a set of homogeneous coordinates of $\mathbb{P}^8$
and $\varphi(s:t) = (s^8 : s^7t : \cdots : t^8)$.
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$$\begin{aligned} H_1 &: T_0 + 4T_2 + 6T_4 + 4T_6 + T_8 &= 0 \\ H_2 &: T_1 - 7T_3 + 7T_5 - T_7 &= 0 \\ H_3 &: T_0 - 14T_2 + 14T_6 - T_8 &= 0. \end{aligned}$$

Put $V = H_1 \cap H_2 \cap H_3$ and $W = H_1 \cap H_2$.
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Example 3

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Theorem 1

Theorem

For a general point P, the Galois group G_P is the full symmetric group S_{d-1} and S_d, corresponding to $P \in C$ and $P \notin C$ respectively.

Now several questions arise:

1. Find Galois points.
2. Find the distribution of the inner and outer Galois points.
3. Find the Galois group G_P at P and the structure of the field K_P.
4. Determine the intermediate fields between $k(\mathbb{P}^1)$ and K_P.
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If $d = 4$, then the number of inner Galois points $\delta(C) = 0, \; 1$ or 4.

The curve with $\delta(C) = 4$ is unique, i.e.,
it is (projectively equivalent to the curve) $y + x^4 + y^4 = 0$.
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If $d \geq 4$, then the number of outer Galois points $\delta'(C) = 0,\ 1$ or 3.

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Proof

The reason that the group is cyclic is as follows:
Suppose $P \in C$ is an inner Galois point.
Then, $\sigma \in G_P$ induces an automorphism of C, since C is smooth.
Moreover, σ is a restriction of a projective transformation, because $d \geq 4$.
Thus we have an injective representation $G_P \hookrightarrow PGL(k, 3)$.
We denote it by the same notation σ.
If ℓ is a line passing through P, then $\sigma(C \cap \ell) = C \cap \ell$.
Taking a suitable coordinates, we can assume that $P = (0 : 0 : 1)$.
Let a_{ij} be the (i, j) component of σ, where $1 \leq i, j \leq 3$, then $a_{13} = a_{23} = 0$, since $\sigma(P) = P$.

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The above research has been generalized as follows:

1. We can consider the Galois point for positive characteristic case.
   In this case we have many different results.
   For some curves there exist an infinitely many Galois points.
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1. We can consider the Galois point for positive characteristic case.
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Example 4

Let us examine the embedding of elliptic curve E associated with the complete linear system $|D|$:

(i) deg $D = 3$ case:

The embedding has a Galois point iff $j(E) = 0$.

$G \cong Z_3$, there exists three Galois points.

In other words, let C be a smooth plane cubic.

Assume $P \in \mathbb{P}^2 \setminus C$ and consider the projection π with the center P to $\mathbb{P}^1$.

Then, π induces a Galois extension $k(C)/k(\pi^*(\mathbb{P}^1))$, or Galois covering

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$X^3 + Y^3 + Z^3 = 0$.
There are three outer Galois points: $(1:0:0)$, $(0:1:0)$ and $(0:0:1)$.
If we use Weierstrass normal form, C is given by
$Y^2Z = 4X^3 + Z^3$ and
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(ii) deg $D = 4$ case:

In this case the embedding has always Galois lines.

$f_D(E) = C \subset \mathbb{P}^3$ has six skew Galois lines

the six lines form a tetrahedron (as in the next page):

and the Galois group $G \cong \mathbb{Z}/2\mathbb{Z} \oplus \mathbb{Z}/2\mathbb{Z}$

If $j(E) = 12^3$, there exist eight Z_4-lines in addition.

In this case the arrangement of Galois lines is very complicated.

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Galois Lines

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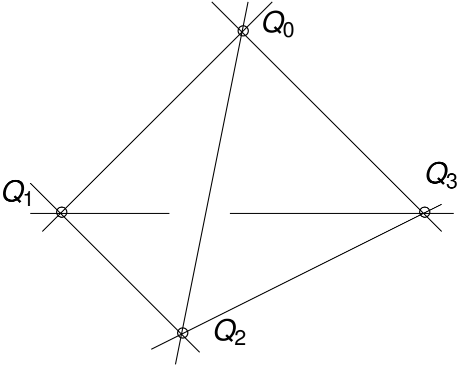

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In other words, let C be a linearly normal smooth genus-one curve in
the projective three space $\mathbb{P}^3$.
Then C has 6 Galois lines ℓ_i ($i = 1, \ldots, 6$)
i.e., the projection with the center ℓ_i to $\mathbb{P}^1$
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